Radiatively Induced Lorentz-Violating Photon Masses

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Abstract

We examine the radiative corrections to an extension of the standard model containing a Lorentz-violating axial vector parameter. At second order in this parameter, the photon self-energy is known to contain terms that violate gauge invariance. Previously, this has been treated as a pathology, but it is also possible to take the gauge noninvariant terms at face value. These terms then make Lorentz-violating contributions to the photon mass, and directly measured limits on the photon mass can be used to set bounds on the Lorentz violation at better than the 10^{-22} GeV level.

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There has recently been a great deal of interest in the possibility of there existing small Lorentz- and CPT-violating corrections to the standard model. Any observed violations of these fundamental symmetries would be important clues about the nature of Planck scale physics. Within the context of effective field theory, a general Lorentz-violating extension of the standard model has been developed [1, 2]. However, the general standard model extension (SME) is extremely complicated, and usually only restricted subsets of the SME (such as the minimal SME, which is gauge invariant and superficially renormalizable) are considered in the literature. The minimal SME provides a framework within which to analyze the results of experiments testing Lorentz violation. To date, such experimental tests have included studies of matter-antimatter asymmetries for trapped charged particles [3, 4, 5, 6] and bound state systems [7, 8], determinations of muon properties [9, 10], analyses of the behavior of spin-polarized matter [11, 12], frequency standard comparisons [13, 14, 15], measurements of neutral meson oscillations [16, 17, 18], polarization measurements on the light from distant galaxies [19, 20, 21], and others.

Radiative corrections to the SME are also a very interesting subject [22]. A great deal of work in this area has concerned the corrections to the electromagnetic Chern-Simons term, with Lagrange density $\mathcal{L}_{CS} = \frac{1}{2}(k_{CS})_\mu \epsilon^{\mu\alpha\beta\gamma} F_{\alpha\beta} A_\gamma$ [19, 23, 24], because $\mathcal{L}_{CS}$ is not gauge invariant (although the related action is). The relevant minimal SME Lagrange density for studying these questions is [22]

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}(i\slashed{\partial} - m - e\slashed{A} - \slashed{b}\gamma_5)\psi. \tag{1}$$

This theory has the potential to induce a finite radiatively-generated Chern-Simons term, with Δk_{CS} proportional to b. However, the coefficient of proportionality depends upon the regularization [25, 26, 27]. Different regulators lead to different values of Δk_{CS}, and through a suitable choice, any coefficient of proportionality between the two may be found. This ambiguity has been extensively studied, and several potentially interesting values of Δk_{CS} have been identified [26, 28, 29, 30, 31, 32, 33]. The ambiguity can be related to the choice of momentum routings in the two triangle diagrams that contribute to Δk_{CS} at leading order. However, the Chern-Simons term is not the only radiative correction in this theory with quite peculiar gauge invariance properties.

The induced Chern-Simons term is a part of the full, Lorentz-violating photon self-energy $\Pi^{\mu\nu}(p)$. However, evaluation of this quantity is tricky. The exact fermion propagator,

$$S(l) = \frac{i}{\slashed{l} - m - \slashed{b}\gamma_5}, \tag{2}$$

may be rationalized to obtain [26, 29]

$$S(l) = i\frac{(\slashed{l} + m - \slashed{b}\gamma_5)(l^2 - m^2 - b^2 + [\slashed{l}, \slashed{b}]\gamma_5)}{(l^2 - m^2 - b^2)^2 + 4[l^2 b^2 - (l \cdot b)^2]}. \tag{3}$$

At $l = 0$, the denominator of the rationalized propagator becomes $(m^2 + b^2)^2$. The square root $|m^2 + b^2|$ of this expression arises in the evaluation of fermion-antifermion

loop diagrams, and the absolute value leads to behavior that is nonanalytic in b. There may be other thresholds for nonanalytic behavior in the photon self-energy as well.

Radiative corrections other than those to the Chern-Simons term are also interesting. We have previously [34, 35] calculated the $\mathcal{O}(e^2 b^2)$ contribution to the photon self-energy in two different regimes—that in which the fermions are massless, $m = 0$; and that in which the mass m is much larger than the scale of the Lorentz violation, or $m^2 \gg |b^2|$. In the latter regime, we also restricted our attention to the zero-momentum case, $p = 0$. In either of these situations, a power series expansion in b is justified, and so it is not too surprising that the same result is found in either case. The $\mathcal{O}(e^2 b^2)$ part of $\Pi^{\mu\nu}$ is

$$\Pi_{b^2}^{\mu\nu} = -\frac{e^2}{24\pi^2} \left(2b^\mu b^\nu + g^{\mu\nu} b^2 \right).$$
(4)

This violates the Ward identity that enforces transversality—$p_\nu \Pi^{\mu\nu}(p) = 0$; however, this result is unambiguous, as there are no momentum routing ambiguities.

Previously, we have treated this result for $\Pi_{b^2}^{\mu\nu}$ as a pathology—as an indication that this method of regulating the theory was inadeqaute. The failure of the Ward identity corresponds to a breakdown of gauge invariance, so this is certainly a reasonable point of view to take. However, it is also possible to consider a different formulation of the theory, one in which the radiative breakdown of gauge symmetry is a real effect. Gauge invariance can only be restored to the theory by introducing a nontrivial regulator for the $\mathcal{O}(e^2 b^2)$ terms in the Feynman diagram expansion. However, while these terms have naive logarithmic divergences, they are unavoidably finite, so the regulator is not strictly necessary. That is, the role of the regulator is no longer to render an infinite result finite, but merely to preserve gauge invariance. We shall therefore dispense with the regulator and take the Ward-identity-violating terms at face value. If this produces a well-defined theory, we may relate our results to experimental tests of special relativity. However, any bounds we find on b will not be as general as those arising from direct measurements of b, because we have considered only one possible version of the theory—one in which the radiative corrections are defined in a particular way.

It is plausible, although unproven, that the result (4) may actually hold for all values of b and m, so long as $p = 0$; this would occur if the nonanalyticities mentioned above turn out not to be a problem at this order in b. However, we shall restrict our attention to the situations in which we know that expanding $\Pi^{\mu\nu}$ as a power series in b is justified. In any case, we expect $m^2 \gg |b^2|$ to represent the physical regime for all charged particles.

In the $m = 0$ case, (4) holds for all values of p. However, in the massive case, we should expect there to be additional momentum-dependent terms that arise for $p \neq 0$. However, terms involving positive powers of p are power-counting finite. These finite terms must automatically obey the Ward identity, unlike the expression (4); they possess the Lorentz structure [36]

$$P^{\mu\nu} = g^{\mu\nu} p^2 b^2 - p^\mu p^\nu b^2 - g^{\mu\nu} (p \cdot b)^2 - p^2 b^\mu b^\nu + (p \cdot b)(p^\mu b^\nu + p^\nu b^\mu).$$
(5)

Just like the usual transverse Lorentz structure $g^{\mu\nu}p^2 - p^\mu p^\nu$ that appears in the photon self-energy, $P^{\mu\nu}$ is proportional to a projector:

$$P^{\mu\rho}P_\rho{}^\nu = [p^2 b^2 - (p \cdot b)^2]P^{\mu\nu}. \tag{6}$$

Because of (6) and the fact that the momentum-dependent terms satisfy the Ward identity, we shall not consider these terms here, although they still might lead to interesting effects. We shall merely note that these terms will not possess poles at $p^2 = 0$. When the full, momentum-dependent self-energy is evaluated at $\mathcal{O}(e^2 b^2)$, with the introduction of a Feynman parameter x, the momentum invariant that appears in the denominator of the expression is the usual $m^2 - x(1-x)p^2$. There cannot therefore be a pole at $p^2 = 0$ unless $m = 0$. However, the full momentum dependence is known in exactly that case, and there is no pole present for $m = 0$ either. So, since they lack any $p^2 = 0$ poles, the momentum-dependent terms will not shift the mass parameter that appears in the photon propagator.

There will be a mass generated by (4), however. We shall demonstrate this by resumming the geometric series of one-particle irreducible diagrams

$$D_{b^2}^{\mu\nu}(p) = \frac{-ig^{\mu\nu}}{p^2} + \frac{-ig^\mu{}_\rho}{p^2}i\Pi_{b^2}^{\rho\sigma}\frac{-ig_\sigma{}^\nu}{p^2} + \frac{-ig^\mu{}_\rho}{p^2}i\Pi_{b^2}^{\rho\sigma}\frac{-ig_{\sigma\kappa}}{p^2}i\Pi_{b^2}^{\kappa\tau}\frac{-ig_\tau{}^\nu}{p^2} + \cdots. \tag{7}$$

The bare photon propagator has been left in the Feynman gauge. Determining $D_{b^2}^{\mu\nu}$ involves the evaluation of the "n-th power" of $\Pi_{b^2}^{\mu\nu}$:

$$\left(\Pi_{b^2}^{(n)}\right)^{\mu\nu} = (\Pi_{b^2})^\mu{}_{\alpha_1}(\Pi_{b^2})^{\alpha_1}{}_{\alpha_2}\cdots(\Pi_{b^2})^{\alpha_{n-1}\nu}, \tag{8}$$

where there are n terms on the right-hand side. This expression is straightforward to evaluate, and we find

$$\left(\Pi_{b^2}^{(n)}\right)^{\mu\nu} = \left(-\frac{e^2 b^2}{24\pi^2}\right)^n\left[(3^n - 1)\frac{b^\mu b^\nu}{b^2} + g^{\mu\nu}\right]. \tag{9}$$

Inserting this into the expansion of $D_{b^2}^{\mu\nu}$ then gives

$$\begin{aligned}
D_{b^2}^{\mu\nu}(p) &= \sum_{n=0}^\infty \frac{-i}{p^2}\frac{\left(\Pi_{b^2}^{(n)}\right)^{\mu\nu}}{(p^2)^n} \tag{10}\\
&= \frac{-i}{p^2}\sum_{n=0}^\infty\left(-\frac{e^2 b^2}{24\pi^2}\frac{1}{p^2}\right)^n\left[3^n\frac{b^\mu b^\nu}{b^2} + \left(g^{\mu\nu} - \frac{b^\mu b^\nu}{b^2}\right)\right]. \tag{11}
\end{aligned}$$

The two Lorentz structures, which project out vectors parallel and perpendicular to b, generate separate geometric series. When summed individually, they give

$$D_{b^2}^{\mu\nu}(p) = -i\frac{b^\mu b^\nu/b^2}{p^2 + \frac{e^2 b^2}{8\pi^2}} - i\frac{g^{\mu\nu} - b^\mu b^\nu/b^2}{p^2 + \frac{e^2 b^2}{24\pi^2}}. \tag{12}$$

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So this does indeed look like a massive theory, although there is not a single mass, but rather two. For the component of A in the same direction as b, the pole in the propagator is shifted to $p^2 = m_\parallel^2 = -\frac{e^2 b^2}{8\pi^2}$, while the mass sqaured parameter for the other components of the gauge field is $m_\perp^2 = -\frac{e^2 b^2}{24\pi^2}$.

These are exactly the two masses that appear in the effective Lagrangian. Including only these gauge-noninvariant radiative corrections, the effective Lagrange density for the purely electromagnetic sector becomes

$$
\begin{aligned}
\mathcal{L}_{b^2} &= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{e^2}{24\pi^2} (A \cdot b)^2 - \frac{e^2 b^2}{48\pi^2} A^2 \qquad (13) \\
&= -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \left[-\frac{e^2 b^2}{8\pi^2} \frac{b^\mu b^\nu}{b^2} - \frac{e^2 b^2}{24\pi^2} \left(g^{\mu\nu} - \frac{b^\mu b^\nu}{b^2} \right) \right] A_\mu A_\nu. \qquad (14)
\end{aligned}
$$

These kinds of Lorentz-violating photon mass operators do not appear in the minimal SME, because they are not gauge invariant. Yet despite the gauge invariance of the underlying bare action, terms of the form $M^{\mu\nu} A_\mu A_\nu$ may yet appear in the effective Lagrangian at one-loop and higher orders.

What we have found looks like a Lorentz-violating (but CPT-preserving) variation of the Proca Lagrangian for massive photons,

$$
\mathcal{L}_{m_\gamma} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m_\gamma^2 A^2. \qquad (15)
$$

A Lorentz-violating mass term more similar to (14) has also been considered in the literature [37, 38]—

$$
\mathcal{L}'_{m_\gamma} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} m_\gamma^2 \vec{A}^2, \qquad (16)
$$

where $\vec{A}^2 = A_j A_j$ is the square of the three-vector potential. (Somewhat more general Lorentz-violating photon mass terms are considered in the appendix.) In $\mathcal{L}'_{m_\gamma}$, there is one component of the field with a different mass parameter. The electrostatic potential A^0 is not coupled to m_γ, and so the Coulomb field is not screened. Bounds of the photon mass are frequently found by observing a lack of screening in the fields of static sources, and so unless these experiments are very carefully formulated, they may really only be constraining the smallest mass parameter in the photon Lagrangian.

In the model we have considered, a spacelike b is favored, since it generates a positive definite mass matrix for the vector degrees of freedom. A timelike b generates tachyonic mass parameters, which render the theory unstable; in the appendix, we show that a timelike b leads to solutions with fields that grow exponentially in time. These instabilities could be prevented by including higher-order gauge-noninvariant terms, such as $(A^2)^2$, because those terms can make large values of the field energetically impossible. Since this theory does not possess a fully gauge-invariant effective action, such terms are not strictly forbidden. However, neither is it clear that there is any mechanism by which

they could arise. For example, any b-dependent contribution to the $\mathcal{O}(e^4)$ four-photon amplitude is finite by power-counting and so will not break gauge symmetry. So without further modifications, the theory will only be stable if b is spacelike or lightlike (in which case, the mass parameters obviously vanish). However, we should point out that even if b is timelike, so that there is an instability, which an $\mathcal{O}(A^4)$ term then stabilizes, the size of the physical mass parameters will still be of order $\frac{e^2 b^2}{24\pi^2}$. (For comparison, a timelike Chern-Simons coefficient also leads to instabilities; however, for the fermionic b itself, a timelike value is actually preferred.)

For a spacelike b, there are indeed physical states with two distinct masses. By mass, we mean precisely the energy eigenvalue at zero momentum, considered in a frame in which b is purely spacelike—$b^\mu = \left(0, \vec{b}\right)$. In the absence of sources, and with all spatial derivatives in the equations of motion vanishing, we have

$$- \partial_0^2 A_j = m_\perp^2 A_j + 2m_\perp^2 \left(\vec{A} \cdot \hat{b}\right) \hat{b}_j, \tag{17}$$

where $\hat{b}$ is the unit vector $\vec{b}/\left|\vec{b}\right|$. The two components of $\vec{A}$ normal to $\vec{b}$ have mass $m_\perp$, as expected, and the parallel component has the larger mass $m_\parallel$. Supplementing (17) is a gauge condition derived from current conservation, $\partial_\mu j^\mu = 0$. The general condition is

$$\partial_\mu A^\mu + 2 \left(\partial_\mu A_\nu\right) \frac{b^\mu b^\nu}{b^2} = 0. \tag{18}$$

In the zero-momentum case we are considering, this becomes $\partial_0 A^0 = 0$, and so it constrains A^0 to be constant in time. Dispersion relations with more general mass terms and nonzero momenta are considered in the appendix.

Within the framework of our model, bounds on the photon mass may be translated into bounds on b. One aspect of this model that is particularly interesting is that it gives us experimental access to the b coefficients for all the charged particles in the theory. Of course, the same thing is true in principle with the radiatively induced Chern-Simons term. However, in that case there are additional complications. Since the Chern-Simons term does not break the gauge invariance of the action, there can be a classical contribution to k_{CS} in addition to the quantum contribution Δk_{CS}. It is impossible to disentangle the classical and radiative parts completely, and this is further complicated by the ambiguity in the radiatively induced part. Moreover, there is good reason to expect that there may be a cancelation among the different species. This can occur if the b terms ultimately arise from the vacuum expectation value of a quantized axial vector field, A_5^μ [2, 30]. Then for each species of fermions, $b^\mu = g\langle A_5^\mu \rangle$, where g is the coupling of that species to A_5^μ. If there is no anomaly associated with the axial vector field, then the anomaly cancelation condition, when multiplied by $\langle A_5^\mu \rangle$, gives

$$\sum_f q_f^2 b_f^\mu = 0. \tag{19}$$

The sum runs over all fermion species (i.e., over the charged leptons and the quarks of all colors and flavors), with q_f being the charge and b_f the Lorentz-violating parameter for a given species. However, the radiatively induced Chern-Simons term is proportional to exactly the quantity $\sum_f q_f^2 b_f$, so it would be no surprise if Δk_{CS} were zero when all contributing particles were considered.

On the other hand, no such cancelation between the species would be expected at second order in b. If the various b_f terms do indeed arise from the expectation value of one common field coupled to all the species of fermions, then all the Lorentz-violating vectors should be aligned. So there are still only two photon mass parameters in the theory,

$$m_\parallel^2 = -\frac{\sum_f q_f^2 b_f^2}{8\pi^2} \tag{20}$$

$$m_\perp^2 = -\frac{\sum_f q_f^2 b_f^2}{24\pi^2}. \tag{21}$$

Even if this hypothesis for the origin of the b parameters is incorrect and these vectors are not collinear, the minimum mass scale induced by the theory should be (barring some special cancelation between the differing parallel and perpendicular components, in the presence of both timelike and spacelike b_f coefficients) roughly the $m_\perp$ given by (21).

The best bounds on the photon mass from direct measurement come from observations made by the Pioneer 10 probe near Jupiter [39]. The upper limit on the mass is 6×10^{-25} GeV. (Solar Probe measurements of the sun's magnetic field could improve this direct bound by at least an order of magnitude [40].) Treating this as a constraint on $m_\perp$ gives

$$0 \leq -\sum_f \left(\frac{q_f}{e}\right)^2 b_f^2 \lesssim 10^{-45} \, (\text{GeV})^2. \tag{22}$$

The lower limit is theoretical, and the upper limit is the experimental one. If the b_f all have a common origin, arising from a single $\langle A_5^\mu \rangle$, then both these limits are completely rigorous. If not, then they are valid again assuming that there are no special cancellations. These limits on b are comparable to or less restrictive than the best limits obtained from direct observations of electrons, muons, and nucleons (with the limits for the proton and neutron translating into bounds for the up and down quarks). However, the bounds given here are much tighter than any present limits on the b coefficients for heavier particles like the tau or heavy quarks. So the result (22) is best interpreted as a rough bound on the scale of b-type Lorentz violation for heavy particles.

There are some much more stringent bounds on the photon mass, taken from studies of fields on astrophysical scales. The strongest limits, which give $m_\gamma \lesssim 3 \times 10^{-36}$ GeV, are based on the properties of magnetic fields on galactic scales [41]. Such limits involve assumptions that may be violated in the presence of Lorentz violation. Limits based on observations of astrophysical plasmas (e.g. the limits found in [42], which are

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conservatively at the $m_\gamma \lesssim 10^{-29}$ GeV level) are likely to be more robost against the effects of Lorentz violation, although there are still questions about these estimates. We shall not use these bounds here, but if they were known to be valid, they would generate correspondingly stronger limits on the b_f.

We should emphasize again that the condition we have found on the fermionic b_f terms is model-dependent in a somewhat subtle way. While the physics of a classical theory is determined entirely by the Lagrangian, the definition of a quantum theory involves additional elements. In particular, we must give a regularization prescription for such a theory. In this case, the model dependence of our limit comes directly from a dependence on how the theory is regulated. In the naive perturbative Feynman diagram expansion, the integral that gives rise to (4) is finite and unambiguous. If no regulator is imposed, then the resulting photon self-energy is not gauge invariant. This is the case that we have been considering. However, it is still possible to introduce a regulator for this integral. Such a regulator would arise naturally if the theory were considered nonperturbatively in b, because in that case, all the $\mathcal{O}(e^2)$ terms in the self-energy would come from the same Feynman diagram; they should therefore all be regulated in the same fashion. If a straightforward dimensional regulator is used, then the nonzero contribution to $\Pi_{b^2}^{\mu\nu}$ (which arises from a surface term) goes away, and the theory is gauge symmetric. In this case, the role of the regulator is not to render an apparently infinite result finite, but only to enforce gauge invariance. So if we do not insist that all the radiative corrections be gauge invariant, then there is no reason to use a regulator for the $\mathcal{O}(e^2 b^2)$ terms, and our limits on the b_f apply. However, whether the theory is regulated in this fashion is ultimately a question that can only be answered experimentally; our limits are then most relevant if we have independent reasons to believe that gauge symmetry may be broken by quantum corrections.

Our calculations have led to a model-dependent bound on a combination of the b_f coefficients for all the charged fermions in the standard model. The bound is based on the fact that radiative corrections involving two powers of these Lorentz-violating parameters may lead to gauge-noninvariant photon mass terms. The bound on the b_f then comes from comparison with experimental constraints on the size of the photon mass. When only directly determined bounds on the photon mass are used, we obtin limits on the b_f at the 10^{-23}–10^{-22} GeV level; these are much better than any other bounds presently existing for the standard model's heavy fermions.

Acknowledgments

Appendix: Waves with Lorentz-Violating Mass Gaps

In this appendix, we derive the dispersion relations for propagating waves when the Lagrangian contains a photon mass interaction of a particular type. A general SME photon mass term will have the form $M^{\mu\nu}A_\mu A_\nu$. In the Proca case, $M^{\mu\nu} = \frac{1}{2}m_\gamma^2 g^{\mu\nu}$. Here, we shall consider more general possibilities, although not the most general symmetric mass matrix. Instead, we shall examine what occurs if there is a frame in which the mass matrix is diagonal, and only one of the four diagonal terms in $M^\mu{}_\nu$ is different from the others, so that there exists a single preferred direction in spacetime. This framework includes all the examples mentioned in this paper as special cases.

The equation of motion in the absence of sources is

$$\partial^\mu F_{\mu\nu} + 2M^\mu{}_\nu A_\mu = 0. \tag{23}$$

Taking a wave Ansatz for the field, $A \propto e^{-ik^\mu x_\mu} = e^{i(\vec{k}\cdot\vec{x} - \omega t)}$, this becomes

$$\left(-\omega^2 + \vec{k}^2\right) A_\nu + k_\nu \left(k^\mu A_\mu\right) + 2M^\mu{}_\nu A_\mu. \tag{24}$$

In addition, there is a gauge-fixing condition that comes from differentiating (23); since $\partial^\mu \partial^\nu F_{\mu\nu} = 0$, the condition $M^\mu{}_\nu \partial^\nu A_\nu = 0$ holds also. We may use this equation to eliminate the $k^\mu A_\mu$ in (24).

Now there are two cases to consider. In the first case, the preferred direction is timelike. With $M^0{}_0 = \frac{1}{2}m_0^2$ and $M^j{}_k = \frac{1}{2}\delta_{jk}m_1^2$, the gauge-fixing condition is

$$m_0^2 \omega A_0 - m_1^2 \vec{k} \cdot \vec{A} = 0. \tag{25}$$

Substituting

$$\omega A_0 = \frac{m_1^2}{m_0^2} \vec{k} \cdot \vec{A} \tag{26}$$

into (24) gives the dispersion relations; these depend on whether $\vec{k}$ and $\vec{A}$ are parallel or perpendicular:

$$\omega^2 = \begin{cases} \vec{k}^2 + m_1^2, & \text{if } \vec{A} \perp \vec{k} \\ \frac{m_1^2}{m_0^2}\vec{k}^2 + m_1^2, & \text{if } \vec{A} \parallel \vec{k} \end{cases}. \tag{27}$$

The time component of the field can then be determined from (26).

The dispersions relations (27) reduce to those of a Lorentz-invariant theory of massive vector particles if $m_0^2 = m_1^2$. However, if $m_0^2 = 0$, as in $\mathcal{L}'_{m_\gamma}$, the expression for the frequency of the longitudinal component becomes singular. Looking back at (25), it is clear that $\vec{k}\cdot\vec{A}$ must actually vanish in this case, so there are only two propagating modes if $m_0^2 = 0$.

The second case, in which rotation invariance is broken, is slightly more complicated. Taking $M^0{}_0 = M^1{}_1 = M^2{}_2 = \frac{1}{2}m_0^2$ and $M^3{}_3 = \frac{1}{2}m_3^2$ (for the radiatively-induced masses discussed in this paper, $m_3 = \sqrt{3}m_0$), the gauge-fixing condition becomes

$$\omega A_0 = \vec{k} \cdot \vec{A} + \left(\frac{m_3^2}{m_0^2} - 1 \right) k_3 A_3. \tag{28}$$

Inserting this into (24) gives a matrix equation that must be diagonalized to obtain the eigenmodes of propagation and the corresponding frequency eigenvalues. This is straightforward to do, but the results are rather unconventional. In fact, the dispersion relations are

$$\omega^2 = \begin{cases} \vec{k}^2 + m_0^2, & \text{if } \vec{A} \perp \hat{e}_3 \\ k_1^2 + k_2^2 + \frac{m_3^2}{m_0^2}k_3^2 + m_3^2, & \text{if } \frac{\vec{A}_\perp}{A_3} = \frac{k_3 \vec{k}_\perp}{m_0^2 + k_3^2} \end{cases}, \tag{29}$$

where $\vec{k}_\perp$ and $\vec{A}_\perp$ are the projections of the vectors $\vec{k}$ and $\vec{A}$ perpendicular to the z-direction, just as k_3 and A_3 are their (scalar) projections along that direction.

The basis of polarization states we have found is not orthogonal. If the mass parameters are negligible compared to the components of $\vec{k}$, then the mode with the unconventional dispersion relation is essentially logitudinally polarized. However, the other modes are not necessarily transverse; their polarization vectors are normal to $\hat{e}_3$, not to $\vec{k}$. In general, a transversely polarized wave will be a superposition of two normal modes with different frequencies.

In either of the two cases considered here, the frequency ω is guaranteed to be real, as long as each of the diagonal terms in $M^\mu{}_\nu$ is positive or zero. However, if any of these mass squared parameters is negative, then there will be values of $\vec{k}$ for which ω is complex, meaning that there are unstable runaway solutions. These solutions grow exponentially with time. For photon masses generated radiatively from b, the instabilities exist if $b^2 > 0$.

Both sets of dispersion relations, (27) and (29), support superluminal propagation, provided the "spacelike" mass parameter m_1 or m_3 is greater than m_0. In that case, the upper limit on achievable speeds in the theory is $\frac{m_1}{m_0}$ or $\frac{m_3}{m_0}$, as appropriate. However, this maximum speed is only approached by longitudinally polarized (or nearly longitudinally polarized) waves. If the coupling to charged matter remains conventional, then all interactions with this superluminal mode are suppressed by powers of the small mass parameters. (Moreover, it is natural to theorize that the results stated in this paragraph can be straightforwardly generalized to any theory with a small, symmetric, positive definite $M^\mu{}_\nu$.) In the timelike case, the propagation speed for longitudinally polarized waves diverges as $m_0^2 \to 0$, and this is related to the fact that the $m_0^2 = 0$ theory possesses an instantaneous Coulomb interaction [37, 38].

References

[1] D. Colladay, V. A. Kostelecký, Phys. Rev. D **55**, 6760 (1997).

[2] D. Colladay, V. A. Kostelecký, Phys. Rev. D **58**, 116002 (1998).

[3] R. Bluhm, V. A. Kostelecký, N. Russell, Phys. Rev. Lett. **79**, 1432 (1997).

[4] R. Bluhm, V. A. Kostelecký, N. Russell, Phys. Rev. D **57**, 3932 (1998).

[5] G. Gabrielse, A. Khabbaz, D. S. Hall, C. Heimann, H. Kalinowsky, W. Jhe, Phys. Rev. Lett. **82**, 3198 (1999).

[6] H. Dehmelt, R. Mittleman, R. S. Van Dyck, Jr., P. Schwinberg, Phys. Rev. Lett. **83**, 4694 (1999).

[7] R. Bluhm, V. A. Kostelecký, N. Russell , Phys. Rev. Lett. **82**, 2254 (1999).

[8] D. F. Phillips, M. A. Humphrey, E. M. Mattison, R. E. Stoner, R. F. C. Vessot, R. L. Walsworth , Phys. Rev. D **63**, 111101 (R) (2001).

[9] R. Bluhm, V. A. Kostelecký, C. D. Lane, Phys. Rev. Lett. **84**, 1098 (2000).

[10] V. W. Hughes, M. G. Perdekamp, D. Kawall, W. Liu, K. Jungmann, G. zu Putlitz, Phys. Rev. Lett. **87**, 111804 (2001).

[11] R. Bluhm, V. A. Kostelecký, Phys. Rev. Lett. **84**, 1381 (2000).

[12] B. Heckel, *et al.*, in *Elementary Particles and Gravitation*, edited by B. N. Kursonoglu, *et al.* (Plenum, New York, 1999).

[13] C. J. Berglund, L. R. Hunter, D. Krause, Jr., E. O. Prigge, M. S. Ronfeldt, S. K. Lamoreaux, Phys. Rev. Lett. **75**, 1879 (1995).

[14] V. A. Kostelecký, C. D. Lane, Phys. Rev. D **60**, 116010 (1999).

[15] D. Bear, R. E. Stoner, R. L. Walsworth, V. A. Kostelecký, C. D. Lane, Phys. Rev. Lett. **85**, 5038 (2000).

[16] V. A. Kostelecký, Phys. Rev. D **61**, 016002 (2000).

[17] Y. B. Hsiung, Nucl. Phys. Proc. Suppl. **86**, 312 (2000).

[18] K. Abe *et al.*, Phys. Rev. Lett. **86**, 3228 (2001).

[19] S. M. Carroll, G. B. Field, R. Jackiw, Phys. Rev. D **41**, 1231 (1990).

[20] S. M. Carroll, G. B. Field, Phys. Rev. Lett. **79**, 2394 (1997).

[21] V. A. Kostelecký, M. Mewes, Phys. Rev. Lett. **87**, 251304 (2001).

[22] V. A. Kostelecký, C. D. Lane, A. G. M. Pickering, Phys. Rev. D **65**, 056006 (2002).

[23] R. Jackiw, S. Templeton, Phys. Rev. D **23**, 2291 (1981).

[24] J. Schonfeld, Nucl. Phys. B **185**, 157 (1981).

[25] R. Jackiw, V. A. Kostelecký, Phys. Rev. Lett. **82**, 3572 (1999).

[26] M. Pérez-Victoria, Phys. Rev. Lett. **83**, 2518 (1999).

[27] M. Pérez-Victoria, JHEP **04**, 032 (2001).

[28] S. Coleman, S. L. Glashow, Phys. Rev. D **59**, 116008 (1999).

[29] J. M. Chung, Phys. Lett. B **461**, 138 (1999).

[30] J. M. Chung, P. Oh, Phys. Rev. D **60**, 067702 (1999).

[31] W. F. Chen, Phys. Rev. D **60**, 085007 (1999).

[32] J. M. Chung, Phys. Rev. D **60**, 127901 (1999).

[33] A. A. Andrianov, P. Giacconi, R. Soldati, JHEP **02**, 030 (2002).

[34] B. Altschul, Phys. Rev. D **69**, 125009 (2004).

[35] B. Altschul, Phys. Rev. D **70**, 101701 (2004).

[36] G. Bonneau, L. C. Costa, J. L. Tomazelli, hep-th/0510045.

[37] G. Gabadadze, L. Grisa, Phys. Lett. B **617**, 124 (2005).

[38] G. Dvali, M. Papucci, M. D. Schwartz, Phys. Rev. Lett. **94**, 191602 (2005).

[39] L. Davis, A. S. Goldhaber, M. M. Nieto, Phys. Rev. Lett. **35**, 1402 (1975).

[40] V. A. Kostelecký, M. M. Nieto, Phys. Lett. B, **317**, 223 (1993).

[41] G. Chibisov, Usp. Fiz. Nauk **19**, 551 (1976); Sov. Phys. Usp. **19**, 624 (1976).

[42] A. Barnes, J. D. Scargle, Phys. Rev. Lett. **35**, 1117 (1975).